Essentially nonnormal numbers for Cantor series expansions

Roman Nikiforov

A real number x is essentially non-normal in base b if no digit of it's b-ary expansion has a relative frequency. The set of essentially nonnormal numbers in base b is known have full Hausdorff dimension and second Baire category. We improve this result of Albeverio, Pratsiovytyi, and Torbin in the following way.

1. We require that no blocks of digits have a relative frequency.

2. All numbers formed by writing digits along any arithmetic progression also have no blocks with a relative frequency.

3. We consider a large class of Cantor series expansions which include the classical b-ary expansions and periodic Cantor series expansions as special cases. This class of bases has full measure for shift invariant ergodic probability measures on $\{2, 3, \ldots\}^{\mathbb{N}}$ satisfying a mild condition.

For basic sequences satisfying our condition, we proved that the set of numbers satisfying the first and second properties has full Hausdorff dimension and second Baire category. Part of the novelty of this proof is the use of recently established properties by Bergelson and Vandehey of continued fraction normal numbers sampled along arithmetic progressions.

The talk is based on joint research with B. Mance.

R. Nikiforov, Dragomanov National Pedagogical University, Kyiv, Ukraine
Adres e-mail: `rnikiforov@gmail.com`