p-ADIC JACQUET-LANGLANDS CORRESPONDENCE AND PATCHING

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Let F be a finite extension of $\mathbb{Q}_p$. The goal of the local p-adic Langlands program is to establish a connection between n-dimensional p-adic Galois representations of the group $\mathrm{Gal}(\bar{F}/F)$ and admissible Banach representations with an action of $\mathrm{GL}_n(F)$. After the initial success of Breuil, Berger, Colmez and others on establishing such a correspondence with desired properties in the case of $n = 2$ and $F = \mathbb{Q}_p$, the progress was hold by different obstacles. The proof of Colmez, purely algebraic in nature, could not be generalized in a straighforward way, due to abundance of automorphic representations for $F \neq \mathbb{Q}_p$ ([BP]). The proof of Harris and Taylor of the classical local Langlands correspondence used crucially geometrical input.

New geometric methods, suitable for p-adic aspects of the Langlands program, became available recently with the rise of perfectoid spaces ([Sch2]). In [Sch1] Scholze gave a construction of certain admissible representations of $D^\times$ (a division algebra) attached to admissible GL_n-representations. He used local geometric methods and exploited the perfectoid structure of the Lubin-Tate space at infinity. Considering his work in two-dimensional case, we are able to extract an admissible 2-dimensional $D^\times$-representation $J'(\rho)$ attached to a local Galois representation $\rho : \mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p) \to \mathrm{GL}_2(E)$, where E is a finite extension of $\mathbb{Q}_p$.

On the other hand, one of us (E.K.) in [Kn], gave a different construction for a possible p-adic Jacquet-Langlands correspondence, by exploiting Drinfeld tower and Cherednik uniformization of Shimura curves. This allowed him to attach an admissible $D^\times$-representation $J(\rho)$ to ρ. Our main theorem is an isomorphism of the two constructions.

We prove it using patching ([CEGGPS1]) and thus our proof is global in nature even though we start with local objects. By using patching and local-global compatibility results we are able to reduce the proof to easy cases when the isomorphism is clear.

References

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